MATH 1551 READING DAY STUDY SESSION WORKSHEET

Problems

1.1 Let $f(x) = \ln(x^2 + 1)$. Determine the domain and range of $f(x)$, and if the function is even or odd.

1.2 True of false: If $g(x)$ is an odd and continuous function defined for all values of x, then we must have $g(0) = 0$.

2.1 True of false: If both $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist, then $\lim_{x\to a} \frac{f(x)}{g(x)}$ must exist.

2.2 Evaluate the following limits: (a) $\lim_{x\to 0} \frac{x(x+3)}{\tan(2x)}$; (b) $\lim_{x\to 3} \frac{x^2-9}{\sqrt{x+1}-2}$.

2.3 True or false: $\lim_{x\to\infty} \frac{\sin x}{x} = 1$.

2.4 Find $\lim\limits_{x\to\infty} \dfrac{4x^2-1}{\sqrt{3x^4+5}}$.

2.5 Give an example of a function f defined on $(-\infty, \infty)$ such that $\lim_{x\to 0^+} f(x)$ exists but $\lim_{x\to 0} f(x)$ doesn't exist.

2.6 If $f(x) = \begin{cases} x^2 + a & x > 1 \\ 2x & x \le 1 \end{cases}$, then how should we choose a in order for f to be a continuous function?

2.7 Find all vertical and horizontal asymptotes of $f(x) = \frac{\sqrt{x}-1}{x^2-x}$.

3.1 Use the definition to compute the derivative of $f(x) = \sqrt{x+1}$.

3.2 For $f(x) = |x^{1/3}|$, is it continuous at $x = 0$? Is it differentiable at $x = 0$?

3.4 Find the derivative of $f(x) = \ln(\tan^{-1}(x^2))$.

3.5 What is the normal line to the graph of $x^2 + xy^4 = 2$ at the point $(1, 1)$?

3.6 A train, starting at 11am, travels east at 45mph while another, starting at noon from the same point, travels south at 30mpg. How fast are they separating at 3pm?

3.7 Approximate the number $\sqrt[4]{1.1}$.

4.1 True or false: Every continuous function must have an absolute maximum in $(-\infty, \infty)$.

4.2 If $f(x) = \frac{x-1}{x^2+1}$, find all intervals such that f is increasing/decreasing.

4.3 Show that $x^5 + e^x = 4$ has exactly one solution.

4.4 True or false: If $f'(x) = 0$ for all x, then $f(x)$ must be a constant.

4.5 True or false: The function $f(x) = \ln(\cos x)$ has a local max at $x = 0$.

4.6 Sketch the graph of $f(x) = x^3 + \frac{3}{x}$.

4.7 Find the dimensions of the rectangle of largest area that can be inscribed in a semicircle of radius 1.

4.8 You are making a square-bottomed box with no top and want to maximize the total volume that it can hold while using no more than 600 square inches of material. What's the biggest box you can make?

4.9 Let $f(x) = x^3 + x - 1$. Use Newton's Method to approximate the value of the x-intercept. Start with $x_0 = 0$ and perform two iterations. (i.e. Find x_2).

4.10 Calculate the antiderivative of $f(x) = \cos x + 3x^3 - \dfrac{2}{1+x^2}$.

Answers

1.1 Even, Domain: (∞, ∞), Range: $[\ln 2, \infty)$.

1.2 True

2.1 False

2.2 (a) $\frac{3}{2}$. (b) 24

2.3 False

2.4 $\frac{4}{\sqrt{3}}$

2.5 one example: $f(x) = \begin{cases} 0 & x \geq 0 \\ 1 & x < 0 \end{cases}$

2.6 $a = 1$

2.7 $y = 0$

3.1 $\frac{1}{2\sqrt{x+1}}$

3.2 Continuous: Yes. Differentiable: No.

3.4 $\frac{2x}{tan^{-1}(x^2)(1+x^4)}$

3.5 $\frac{4}{3}x - \frac{1}{3}$

3.6 $24\sqrt{5}$ mph

3.7 ≈ 1.025

4.1 False

4.2 Increasing in $(1-\sqrt{2}, 1+\sqrt{2})$ and decreasing in $(-\infty, 1-\sqrt{2})$ and $(1+\sqrt{2}, \infty)$.

4.3 f is continuous. $f(0) < 4$ and $f(2) > 4$, so by IVT there exists at least one solution in $(0, 2)$. $f' > 0$ for all x implies no critical points, so by Rolle's Theorem at most one solution. Hence, exactly one solution.

4.4 True

4.5 True

4.6 -

4.7 $\sqrt{2} \times \frac{1}{\sqrt{2}}$

4.8 Maximum volume: $1000\sqrt{2}$

4.9 $\frac{3}{4}$

4.10 $\sin x + \frac{3}{4}x^4 - 2\tan^{-1} x + C$