MATH 2550/2551 READING DAY STUDY SESSION WORKSHEET

Problems marked with ** are only relevant for MATH 2551.

PROBLEMS

1. Find $T$, $N$, and curvature for $r(t) = (3 \sin t)\mathbf{i} + (3 \cos t)\mathbf{j} + 4t\mathbf{k}$.

2. **Write acceleration in terms of tangential and normal components for $r(t) = (t + 1)\mathbf{i} + 2t\mathbf{j} + t^2\mathbf{k}$, $t = 1$.

3. Find the limit: $\lim_{{(x,y) \to (0,0)}} \frac{3x^2 - y^2 + 5}{x^2 + y^2 + 2}$.

4. Find all second-order partial derivatives for $w(x, y) = x \sin(x^2y)$.

5. Evaluate $\frac{dw}{dt}$ for $w = 2ye^x - \ln z$, $x = \ln(t^2 + 1)$, $y = \arctan t$, $z = e^t$, $t = 1$.

6. Find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ for $z^3 - xy + yz + y^3 - 2 = 0$, $(1, 1, 1)$.

7. Find the derivative of the function at the point in the direction of $\mathbf{v}$ for $f(x, y) = 2xy - 3y^2$, $P = (5, 5)$, $\mathbf{v} = 4\mathbf{i} + 3\mathbf{j}$.

8. Find the tangent plane and normal line for $2z - x^2 = 0$, $P_0(2, 0, 2)$.

9. Find all local maxima, minima, and saddle points for $f(x, y) = x^2 + xy + y^2 + 3x - 3y + 4$.

10. Find absolute maxima and minima for $f(x, y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangular plate bounded by the lines $x = 0$, $y = 2$, $y = 2x$ in the first quadrant.

11. Find the maximum value of $f(x, y) = 49 - x^2 - y^2$ on the line $x + 3y = 10$.

12. Compute the area bounded by $y = \sqrt{x}$, $y = 0$, and $x = 9$ by an iterated integral using (a) vertical cross-sections and (b) horizontal cross-sections.

13. Sketch the region and evaluate $\int_0^1 \int_0^{y^2} 3y^3e^{xy} dxdy$.

14. Change to polar and evaluate $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} dydx$.

15. For the region below $z = 4 - xy$ and in $0 \leq x \leq 2$, $0 \leq y \leq 1$, integrate the function $3 - 4x$.

16. Use $x = \frac{u}{v}$, $y = uv$ in $\mathbb{R}$ (first quadrant) bounded by $xy = 1$, $xy = 9$, $x = y$, $y = 4x$ for $\int \int \left( \frac{y}{x} + \sqrt{xy} \right) dxdy$.

17. **Evaluate the line integral $\int (xy + z)ds$ along the curve $r(t) = 2t\mathbf{i} + t^2\mathbf{j} + (2 - 2t)\mathbf{k}$, $0 \leq t \leq 1$.

18. **Find the flow of the field $F = -4xy\mathbf{i} + y8\mathbf{j} + 2\mathbf{k}$ along $r(t) = ti + t^2\mathbf{j} + \mathbf{k}$, $0 \leq t \leq 2$.

19. **Find the circulation and flux of the field $F = x\mathbf{i} + y\mathbf{j}$ around $r(t) = \cos t\mathbf{i} + \sin t\mathbf{j}$, $0 \leq t \leq 2\pi$.

20. **Find the potential function for $F = e^{y+2z}(i + x\mathbf{j} + 2x\mathbf{k})$.

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21. **Use Green’s Theorem to find counterclockwise circulation and outward flux for field \( F = (y^2 - x^2)i + (x^2 + y^2)j \) and curve \( C: y = 0, x = 3, y = x \).

22. **Use a parametrization to express the area of the surface as a double integral: the portion of the cone \( z = 2\sqrt{x^2 + y^2} \) between the planes \( z = 2 \) and \( z = 6 \).

23. Evaluate \( \int \int 2y \, dV \) over \( S: y^2 + z^2 = 4 \) between \( x = 0, x = 3 - z \).

24. At time \( t = 0 \), a particle is located at the point \( (0, 1, 2) \). At this time it is traveling towards the point \( (3, 2, 3) \), has speed 3 at \( (0, 1, 2) \), and has constant acceleration \( 3i - j + k \). Find an equation for the position vector \( \vec{r}(t) \) at time \( t \).

25. Let \( P(A_0, B_0, C_0), Q(A_1, B_1, C_1) \) be two distinct points in 3-dimensional space.
   (a) Write down a vector parameterization for the line \( PQ \).
   (b) Show that the curvature \( \kappa \) of \( PQ \) is 0.
   (c) **Show that the torsion \( \tau \) of \( PQ \) is 0.

26. (a) At what points \( (x, y) \) in the plane is the function \( f(x, y) = \frac{y}{1 + \cos x} \) continuous?
   (b) Find the following limit by first rewriting the fraction \( \lim_{(x,y) \to (1,-1)} \frac{x^3 + y^3}{x + y} \).

27. Let \( C \) be the smooth curve given by the intersection of the two surfaces \( xyz = 1 \) and \( x^2 + 2y^2 + 3z^2 = 6 \).
   (a) Write down an equation that describes \( C \) implicitly.
   (b) Write down a parametric equation for the line tangent to \( C \) at the point \( (1, 1, 1) \).

28. Let \( D \) be the cylinder bounded below by \( z = -1 \), bounded on the sides by \( x^2 + y^2 = 1 \), and bounded above by \( z = 1 \). (Do not evaluate the integrals.)
   (a) Express the volume of \( D \) as an iterated triple integral in cylindrical coordinates.
   (b) Express the volume of \( D \) as an iterated triple integral in rectangular coordinates.
   (c) Express the volume of \( D \) as an iterated triple integral in spherical coordinates.

29. Consider the function \( f(x, y) = \frac{\ln(x^2 + y^2)}{\sqrt{x^2 + y^2}} \).
   (a) Describe the region \( R \) given by \( 1 \leq x^2 + y^2 \leq e \) in polar coordinates.
   (b) Write down the integral of \( f(x, y) \) over \( R \) in polar coordinates.
   (c) Evaluate the integral you found in part (b).

30. Let \( D \) be the region in \( xyz \)-space defined by the inequalities \( 1 \leq x \leq 2, 0 \leq xy \leq 2, 0 \leq z \leq 1 \). Consider the coordinate transformation of \( D \) to the \( uvw \)-plane given by \( u = x, v = xy, w = 3z \).
   (a) Sketch the preimage \( G \) of \( D \) under the coordinate transformation in the \( uvw \)-plane and label the bounding curves.
   (b) Write down the Jacobian associated to this coordinate transformation.
   (c) Evaluate the integral \( \iiint_D (x^2y + 3xyz) \, dx \, dy \, dz \).

31. A flat circular plate has the shape of the region \( x^2 + y^2 \leq 1 \). Points on the plate have temperature \( T(x, y) = x^2 + 2y^2 - x \). Find the temperatures of the hottest and coldest points of the plate.

32. Find the points on the surface \( xyz = 1 \) closest to the origin.

33. Find the volume of the wedge cut from the cylinder \( x^2 + y^2 \leq 1 \) by the planes \( z = -y \) and \( z = 0 \).
1. \( T = \frac{3 \cos t}{5} \vec{i} - \frac{3 \sin t}{5} \vec{j} + \frac{4}{5} \vec{k} \), \( N = (-\sin t) \vec{i} - (\cos t) \vec{j}, \cdot \kappa = \frac{3}{25} \)

2. \( a(1) = \frac{4}{3} T + \frac{2 \sqrt{5}}{3} N \)

3. \( \frac{5}{2} \)

4. \( w_{xx} = 6xy \cos(x^2y) - 4x^3y^2 \sin(x^2y), w_{yy} = -x^5 \sin(x^2y), w_{xy} = 3x^2 \cos(x^2y) - 2x^4y \sin(x^2y) \)

5. \( \frac{dw}{dt} = 4t \arctan t + 1 = \pi + 1 \)

6. \( \frac{1}{4}, -\frac{3}{4} \)

7. \(-4\)

8. \( 2x - z - 2 = 0, r(t) = (2, 0, 2) + t(-4, 0, 2) \)

9. \( f(-3, 3) = -5 \) minimum

10. \( (0, 0) = 1, \text{ abs max, } (1, 2) = -5, \text{ abs min} \)

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12. (a) Vertical: \( \int_0^1 \int_0^{\sqrt{x}} dy \, dx = 18 \); (b) Horizontal: \( \int_0^3 \int_0^9 dxdy = 18 \)

13. \( e - 2 \)

14. \( \frac{5}{2} \)

15. \( \int_0^1 \int_0^{1-x} (3 - 4x) \, dy \, dx = -\frac{17}{3} \)

16. \( \int_1^2 \int_1^3 \frac{(u+v)^2u}{v} \, du \, dv = 8 + \frac{52}{3} \ln 2 \)

17. \( \frac{13}{2} \)

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19. Circulation = 0; Flux = 2\pi

20. \( f(x, y, z) = xe^{y+2z} + C \)

21. Flux = -9, Circulation = 9

22. \( \int_0^{2\pi} \int_1^3 r \sqrt{5} r \, dr \, d\theta \)

23. \( r(u, v) = r(x, \theta) = (x, 2 \sin \theta, 2 \cos \theta) \)

\[
\iint f(r(u, v)) |r_u \times r_v| \, dA = \int_0^{2\pi} \int_0^{3-2\cos \theta} 2(2 \sin \theta)(2) \, dx \, d\theta = 0
\]

24. \( \vec{r}(t) = \left( \frac{3t^2}{2} + \frac{9}{\sqrt{11}} \right) \vec{i} + \left( -\frac{1}{2}t^2 + \frac{3}{\sqrt{11}}t + 1 \right) \vec{j} + \left( \frac{1}{2}t^2 + \frac{3}{\sqrt{11}}t + 2 \right) \vec{k} \)

25. (a) \( L(t) = (x(t) = A_0 + (A_1 - A_0)t, y(t) = B_0 + (B_1 - B_0)t, z(t) = C_0 + (C_1 - C_0)t) \)

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26. (a) \( f \) is continuous on \( \{(x, y) | x \neq (2k + 1)\pi; k = \ldots, -1, 0, 1, \ldots \} \). (b) 1

27. (a) \( x^2 + 2y^2 + 3z^2 = 6xyz \). (b) \( L(t) = (1 + 2t, 1 - 4t, 1 + 2t) \)

28. (a) \( \int_{-1}^{1} \int_{0}^{2\pi} r \, d\theta \, dz \). (b) \( \int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} z \, dx \, dy \)
   (c) \( \int_{0}^{2\pi} \int_{0}^{\pi/4} \int_{0}^{1} \rho^2 \sin \varphi \, d\varphi \, d\rho \, d\theta + 2 \int_{0}^{2\pi} \int_{0}^{\pi/4} \int_{0}^{1} \cos \varphi \rho^2 \sin \varphi \, d\varphi \, d\rho \, d\theta \)

29. (a) \( 1 \leq r^2 \leq e \). (b) \( \int_{0}^{\pi} \int_{1}^{\sqrt{e-1}} 2 \ln(r) \, dr \, d\theta \). (c) \(-2\pi \sqrt{e} + 4\pi \)

30. (b) \( \frac{1}{3\sqrt{e}} \). (c) \( 2 + 3 \ln(2) \)

31. Hottest: \( \frac{9}{4} \) at points \( (-\frac{1}{2}, \sqrt{\frac{3}{2}}), (-\frac{1}{2}, -\sqrt{\frac{3}{2}}) \). Coldest: \( -\frac{1}{4} \) at point \( (\frac{1}{2}, 0) \).

32. Points \( (1,1,1), (-1,-1,-1), (1,-1,-1), \) and \( (-1,1,-1) \) are all at distance 3 from the origin.

33. \( \frac{2}{3} \)