MATH 2551 Multivariable Calculus Final Review Fall 2017

1. Find T, N and curvature for $r(t) = (3\sin t)i + (3\cos t)j + 4tk$
2. Write acceleration in terms of tangential and normal components for $r(t) = (t+1)i + 2tj + t^2k, t = 1$
3. Find the limit: $\lim_{(x,y)\to(0,0)} \frac{3x^2-y^2+5}{x^2+y^2+2}$
4. Find all second-order partial derivatives for $w = x\sin(x^2y)$
5. Evaluate $\frac{dw}{dt}$ for $w = 2ye^x - \ln z, x = \ln(t^2+1), y = \arctan t, z = e^t, t = 1$
6. Find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ for $z^3 - xy + yz + y^3 - 2 = 0, (1,1,1)$
7. Find the derivative of the function at the point in the direction of *v* for $f(x,y) = 2xy - 3y^2, P = (5,5), v = 4i + 3j$
8. Find the tangent plane and normal line for $2z - x^2 = 0, P_0(2,0,2)$
9. Find all local maxima, minima, and saddle points for $f(x,y) = x^2 + xy + y^2 + 3x - 3y + 4$
10. Find absolute maxima and minima for $f(x,y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangular plate bounded by the lines $x = 0$, $y = 2$, $y = 2x$ in the first quadrant
11. Find the maximum value of $f(x,y) = 49 - x^2 - y^2$ on the line $x + 3y = 10$
12. Write an iterated integral using (a) vertical cross-sections and (b) horizontal cross-sections for $y = \sqrt{x}, y = 0, x = 9$
13. Sketch the region and evaluate $\int_0^1 \int_0^{y^2} 3y^3e^{xy}\,dxdy$
14. Change to polar and evaluate $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} dydx$
15. For the region below $z = 4 - xy$ and in $0 \le x \le 2, 0 \le y \le 1$ integrate the function $3 - 4x$
16. Use $x = \frac{u}{v}, y = uv$ in R (first quadrant) bounded by $xy = 1, xy = 9, y = x, y = 4x$ for $\iint \left(\sqrt{\frac{y}{x}} + \sqrt{xy}\right) dx\,dy$
17. Evaluate the line integral $\int (xy + y + z)\,ds$ along the curve $r(t) = 2ti + tj + (2-2t)k, 0 \le t \le 1$
18. Find the flow of the field $F = -4xyi + 8yj + 2k$ along $r(t) = ti + t^2 + k, 0 \le t \le 2$
19. Find the circulation and flux of the field $F = xi + yj$ around $r(t) = \cos t\, i + \sin t\, j, 0 \le t \le 2\pi$
20. Find the potential function for $F = e^{y+2z}(i + xj + 2xk)$
21. Use Green's Theorem to find counterclockwise circulation and outward flux for field F and curve C: $F = (y^2 - x^2)i + (x^2 + y^2)j, C: y = 0, x = 3, y = x$
22. Use a parametrization to express the area of the surface as a double integral: the portion of the cone $z = 2\sqrt{x^2 + y^2}$ between the planes $z = 2$ and $z = 6$
23. Evaluate $\iint 2y\,dV$ over S: $y^2 + z^2 = 4$ between $x = 0, x = 3 - z$

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1. $T = \frac{3\cos t}{5} i - \frac{3\sin t}{5} j + \frac{4}{5} k,\;\; N = (-\sin t)i - (\cos t)j,\;\; \kappa = \frac{3}{25}$
2. $a(1) = \frac{4}{3} T + \frac{2\sqrt{5}}{3} N$
3. 5/2
4. $w_{xx} = 6xy\cos(x^2y) - 4x^3y^2\sin(x^2y)\,, w_{yy} = -x^5\sin(x^2y)\,, w_{xy} = 3x^2\cos(x^2y) - 2x^4\,y\sin(x^2y)$
5. $\frac{dw}{dt} = 4t\arctan t + 1 = \pi + 1$
6. $\frac{1}{4}, -\frac{3}{4}$
7. -4
8. $2x - z - 2 = 0, r(t) = (2,0,2) + t(-4,0,2)$
9. $f(-3,3) = -5$ minimum
10. $(0,0) = 1, abs\ max, (1,2) = -5, abs\ min$
11. 39
12. $0 \le x \le 9, 0 \le y\sqrt{x}, 0 \le y \le 3, y^2 \le x \le 9$
13. $e - 2$
14. $\frac{\pi}{2}$
15. $\int_0^2 \int_0^1 \int_0^{4-xy} (3 - 4x)\,dz\,dy\,dx = -\frac{17}{3}$
16. $\int_1^2 \int_1^3 \frac{(u+v)2u}{v}\,du\,dv = 8 + \frac{52}{3}\ln 2$
17. 13/2
18. 48
19. Circulation = 0; Flux = 2π
20. $f(x,y,z) = xe^{y+2z} + C$
21. $Flux = -9, circ = 9$
22. $\int_0^{2\pi} \int_1^3 r\sqrt{5}\,dr\,d\theta$
23. $r(u,v) = r(x,\theta) = \langle x, 2\sin\theta\,, 2\cos\theta\rangle$,

$$\iint f(r(u,v))|r_u \times r_v|\,dA = \int_0^{2\pi} \int_0^{3-2\cos\theta} 2(2\sin\theta)(2)\,dx\,d\theta = 0$$